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THE LOCALIZATION OF PLASTIC DEFORMATION IN THERMOPLASTIC SOLIDS

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Abstract—The main objective of this paper is the investigation of the influence of thermomechanical couplings and thermal softening effects on adiabatic shear band localization criteria for finite rate-independent deformation of an elastic–plastic body. The constitutive equations for thermoelastic–plastic J_2-flow theory are formulated within a framework of the rate type covariance structure with internal state variables. Two alternative descriptions are presented. Both constitutive structures formulated are invariant with respect to diffeomorphisms and are materially isomorphic. Particular attention is focused on the coupling phenomena generated by the internal heat resulting from internal dissipation. An identification procedure has been developed which permits the determination of the exact form of the evolution equation for the internal state variable vector. A set of coupled evolution equations for the Kirchhoff stress tensor and for temperature is investigated. The assumption that the thermodynamic process considered is adiabatic permits the elimination of the rate of temperature and gives the fundamental evolution equation for the Kirchhoff stress tensor. This important result allows the use of the standard bifurcation method in the examination of the adiabatic shear band localization criteria. For the particular elastic properties of the material and for some simplified case of the coupling effects the criteria for adiabatic shear band localization are obtained in exact analytical form. Discussions of the influence of thermomechanical couplings, thermal expansion, thermal plastic softening effects and the covariance terms on the localization criteria are presented.

1. INTRODUCTION

In recent years there has been active research work in the field of the instability phenomena of plastic flow processes. Particularly the localization of plastic deformation along a shear band treated as a prelude to failure initiation has been a matter of great interest.

It has been shown that the onset of localization does depend critically on the assumed constitutive law.

Rice (1976) in his fundamental work on the localization of plastic deformation wrote: "The present study shows that conditions for localization relate closely to subtle and not well understood features of the constitutive description of plastic flow... While the constitutive modelling of these features needs to be improved in relation to the detailed mechanisms of deformation, so also is there need for a fuller assessment of the role of imperfections or initial non-uniformities in material properties in promoting localization. Indeed, the latter approach seems mandatory for rate-dependent plastic flow models and these, as well as the range of thermomechanically coupled localization phenomena, would seem to merit further study."

In the mean time different constitutive features have been analysed and their influence on the onset of localization have been investigated.

Particular attention has been focused on the following effects:

(i) yield surface vertices (cf. Rudnicki and Rice, 1975; Needleman and Rice, 1978);

(ii) deviation from plastic "normality", i.e. deviation from an associated flow rule (cf. Rudnicki and Rice, 1975; Needleman and Rice, 1978; Rice and Rudnicki, 1980; Duszek and Perzyna, 1988a,b);

(iii) the dilatational plastic flow due to nucleation and growth of microvoids (cf. Rudnicki and Rice, 1975; Needleman and Rice, 1978; Duszek and Perzyna, 1988a,b);

(iv) strain-induced anisotropy modelled as the kinematic hardening rule (cf. Mear and Hutchinson, 1985; Tvergaard, 1987; Duszek and Perzyna, 1988a,b);

(v) influence of the covariance terms (co-rotational terms) (cf. Rudnicki and Rice, 1975; Rice and Rudnicki, 1980; Lippmann, 1986).

The investigation of the influence of thermomechanical coupling effects on the localization phenomenon needs greater attention.

In many technological processes such as plastic shaping and forming, low temperature processes, dynamic fragmentation and high velocity machining thermal effects may have a significant influence on the localization phenomenon.

Thermal effects may have greater influence on formation of shear bands in dynamic loading processes when the heat that is produced during plastic deformation is given insufficient time to be conducted away. Then the process considered is adiabatic and localization occurs more readily.

Experimental results which confirmed such conjecture have been recently reported by Hartley *et al.* (1987), Marchand and Duffy (1988) and Marchand *et al.* (1988).

The experimental investigations of Hartley *et al.* (1987) were performed under dynamic loading conditions for AISI 1018 cold rolled steel and AISI 1020 hot rolled steel. They observed that the formation of shear bands for both steels is influenced greatly by thermal effects. They showed that even a small increase of local temperature (about 50–80 C) for 1018 CRS steel enhances local deformation and local heating, and as a result, causes initiation of shear band formation. A similar result has been reported for a low alloy structural steel (HY-100) by Marchand and Duffy (1988).

The main objective of this paper is the investigation of the influence of thermomechanical couplings and thermal softening effects on adiabatic shear band localization criteria for finite rate-independent deformation of an elastic–plastic body.

In Section 2 a general internal state variable framework is presented. Particular attention is focused on the discussion of the spatial covariance constitutive structure.

In Section 3 the constitutive equations for thermoelastic–plastic J_2-flow theory are formulated within a framework of the rate type covariance structure with internal state variables. Two alternative descriptions of thermomechanical couplings for J_2-flow theory are presented. In the first the Lie derivative is used to define the objective rate of the Kirchhoff stress tensor, while in the second the Zaremba–Jaumann rate is utilized. Both constitutive structures formulated are invariant with respect to diffeomorphisms and are materially isomorphic.

Attention is focused on the coupling phenomena generated by the internal heat resulting from internal dissipation. By applying the Legendre transformation and a careful analysis of the internal dissipation during the plastic flow process the identification procedure is developed. This method of identification permits the determination of the exact form of the evolution equation for the internal state variable vector which is responsible for the dissipative nature of plastic flow phenomena.

Section 4 is devoted to the investigation of the adiabatic process. A set of coupled evolution equations for the Kirchhoff stress tensor and for temperature is investigated. A method has been developed which allows us to obtain the fundamental rate criterion for the Kirchhoff stress tensor. The matrix in this equation describes all thermomechanical couplings introduced. This important result allows us to use the standard bifurcation method in the examination of the adiabatic shear band localization criteria.

The main contribution to thermomechanical couplings has been carefully discussed. Based on this analysis, the simplified evolution equation for the temperature is obtained.

The predictions, by applications of the localization criterion, are given in Section 5.

A procedure has been developed which allows us to discuss two separate new effects on the localization phenomenon along a shear band. One is thermomechanical coupling when there is no spatial covariance effect and the other is the spatial covariance effect for an assumed isothermal process. For both cases the criteria for adiabatic shear band localization are obtained in exact analytical form.

The influence of two important thermal effects, namely thermal expansion and thermal plastic softening, on the criteria for localization of plastic deformation is investigated. The similar influence of spatial covariance terms is also examined. A discussion of the results obtained is presented in Section 6.

2. INTERNAL STATE VARIABLE FRAMEWORK

2.1. *Basic assumptions and definitions*

Let us assume that a continuum body is an open bounded set $\mathscr{B} \subset \mathbb{R}^3$, and let $\phi : \mathscr{B} \to \mathscr{S}$ be a C^1 configuration of $\mathscr{B}$ in $\mathscr{S}$. The tangent of ϕ is denoted $\mathbf{F}$ and is called the deformation gradient of ϕ.

Let $\{X^A\}$ and $\{x^a\}$ denote coordinate systems on $\mathscr{B}$ and $\mathscr{S}$, respectively. Then we refer to $\mathscr{B} \subset \mathbb{R}^3$ as the reference configuration of a continuum body with particles $\mathbf{X} \in \mathscr{B}$ and to $\mathscr{S} = \phi(\mathscr{B})$ as the current configuration with points $\mathbf{x} \in \mathscr{S}$. The matrix $\mathbf{F}(\mathbf{X}, t) = \partial\phi(\mathbf{X}, t)/\partial\mathbf{X}$ with respect to the coordinate bases $\mathbf{E}_A(\mathbf{X})$ and $\mathbf{e}_a(\mathbf{x})$ is given by

$$F^a_A(\mathbf{X}, t) = \frac{\partial \phi^a}{\partial X^A}(\mathbf{X}, t), \tag{1}$$

where a mapping $\mathbf{x} = \phi(\mathbf{X}, t)$ represents a motion of a body $\mathscr{B}$.

In a neighbourhood of $\mathbf{X}$, i.e. in $\mathscr{N}(\mathbf{X})$ for every $\mathbf{X} \in \mathscr{B}$ we consider the local multiplicative decomposition

$$\mathbf{F} = \mathbf{F}^e \cdot \mathbf{F}^p, \tag{2}$$

where $(\mathbf{F}^e)^{-1}$ is the deformation gradient that releases elastically the stress on the neighbourhood $\phi(\mathscr{N}(\mathbf{X}))$ in the current configuration.

We define the total and elastic Finger deformation tensors

$$\mathbf{b} = \mathbf{F} \cdot \mathbf{F}^T, \quad \mathbf{b}^e = \mathbf{F}^e \cdot \mathbf{F}^{eT}, \tag{3}$$

respectively, and the Eulerian strain tensors as follows:

$$\mathbf{e} = \tfrac{1}{2}(\mathbf{g} - \mathbf{b}^{-1}), \quad \mathbf{e}^e = \tfrac{1}{2}(\mathbf{g} - \mathbf{b}^{e\,-1}), \tag{4}$$

where $\mathbf{g}$ denotes the metric tensor in the current configuration.

By definition

$$\mathbf{e}^p = \mathbf{e} - \mathbf{e}^e = \tfrac{1}{2}(\mathbf{b}^{e\,-1} - \mathbf{b}^{-1}) \tag{5}$$

we introduce the plastic Eulerian strain tensor.

In many fields of mechanics and particularly in continuum mechanics the Lie derivative plays a very important role.† The Lie derivative of a spatial tensor field $\mathbf{t}$ with respect to the velocity field $\mathbf{v}$ can be defined as

$$L_{\mathbf{v}}\mathbf{t} = \phi_* \frac{\partial}{\partial t}(\phi^* \mathbf{t}), \tag{6}$$

where ϕ^* and ϕ_* denote the pull-back and push-forward operations, respectively.

We define the rates of deformation as follows:

$$\begin{aligned} \mathbf{d} &= L_{\mathbf{v}}\mathbf{e} = \tfrac{1}{2}L_{\mathbf{v}}\mathbf{g} \\ d_{ab} &= \tfrac{1}{2}(L_{\mathbf{v}}\mathbf{g})_{ab} = \tfrac{1}{2}(g_{ac}v^c|_b + g_{cb}v^c|_a), \end{aligned} \tag{7}$$

† For precise definition of the Lie derivative and its algebraic and dynamic interpretations please consult Abraham *et al.* (1988). Applications of the Lie derivative to theoretical mechanics may be found in Abraham and Marsden (1978) and to continuum mechanics in Marsden and Hughes (1983).

where

$$v^a|_b = \frac{\partial v^a}{\partial x^b} + \gamma^a_{bc} v^c \tag{8}$$

and γ^a_{bc} denotes the Christoffel symbol for the general coordinate system $\{x^a\}$.

Similarly the rate of plastic deformation is given by

$$\mathbf{d}^p = L_v \mathbf{e}^p = \tfrac{1}{2} L_v(\mathbf{b}^{e\,-1}) \tag{9}$$

and

$$\mathbf{d} = \mathbf{d}^e + \mathbf{d}^p. \tag{10}$$

For any scalar function we have

$$L_v f = \dot{f} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^a} v^a. \tag{11}$$

Let τ denote the Kirchhoff stress tensor related to the Cauchy stress tensor σ by

$$\tau = J\sigma = \frac{\rho_{\mathrm{Ref}}}{\rho}\sigma, \tag{12}$$

where $\rho_{\mathrm{Ref}}(\mathbf{X})$ and $\rho(\mathbf{x}, t)$ denote the mass density in the reference and current configuration, respectively, and the Jacobian J is the determinant of the linear transformation $\mathbf{F}(\mathbf{X}, t) = (\partial/\partial \mathbf{X})\phi(\mathbf{X}, t)$.

It is noteworthy that any possible objective rate of the stress tensor is a particular case of the Lie derivative (cf. Marsden and Hughes, 1983).

The Lie derivative of the Kirchhoff stress tensor τ (provided we have only contravariant coordinate representation τ^{ab} in mind) gives

$$\begin{aligned} L_v \tau &= \phi_* \frac{\partial}{\partial t}(\phi^* \tau) = \left\{ \mathbf{F} \cdot \frac{\partial}{\partial t}[\mathbf{F}^{-1} \cdot (\tau \circ \phi) \cdot \mathbf{F}^{-1\mathrm{T}}] \cdot \mathbf{F}^{\mathrm{T}} \right\} \circ \phi^{-1} \\ &= \dot{\tau} - (\mathbf{d} + \omega) \cdot \tau - \tau \cdot (\mathbf{d} + \omega)^{\mathrm{T}}, \end{aligned} \tag{13}$$

where $\circ$ denotes the composition of mappings and ω the spin tensor is defined as follows

$$\omega_{ab} = \tfrac{1}{2}(g_{ac} v^c|_b - g_{cd} v^c|_a). \tag{14}$$

In the coordinate system (13) reads

$$\begin{aligned} (L_v \tau)^{ab} &= F^a_A \frac{\partial}{\partial t}(F^{-1A}_c \tau^{cd} F^{-1B}_d) F^b_B \\ &= \frac{\partial \tau^{ab}}{\partial t} + \frac{\partial \tau^{ab}}{\partial x^c} v^c - \tau^{cb} \frac{\partial v^a}{\partial x^c} - \tau^{ac} \frac{\partial v^b}{\partial x^c}. \end{aligned} \tag{15}$$

Equation (15) defines the Oldroyd rate of the Kirchhoff stress tensor τ (cf. Oldroyd, 1950).

The Zaremba–Jaumann stress rate or the co-rotated derivative of the Kirchhoff stress tensor τ is defined as

$$\overset{\mathrm{v}}{\tau} = \dot{\tau} - \omega \cdot \tau + \tau \cdot \omega. \tag{16}$$

Comparison of (16) with (13) gives the result

$$L_v\boldsymbol{\tau} = \overset{\nabla}{\boldsymbol{\tau}} - \mathbf{d}\cdot\boldsymbol{\tau} - \boldsymbol{\tau}\cdot\mathbf{d}. \tag{17}$$

The result (17) will be used in the formulation of the alternative constitutive descriptions of thermoelastic–plastic responses of solids.

2.2. *Objective constitutive structure*

To describe the dissipation phenomena during a thermoplastic flow process we have to introduce the internal state vector $\boldsymbol{\mu} \in V_n$, where V_n is n-dimensional vector space.

The intrinsic state of a particle $\mathbf{X} \in \mathscr{B}$ at time t is determined by a set of variables

$$\mathfrak{s} = (\mathbf{e}, \mathbf{F}, \vartheta; \boldsymbol{\mu}), \tag{18}$$

where $\mathbf{e}$ is the Eulerian strain tensor, $\mathbf{F}$ is the deformation gradient, ϑ denotes absolute temperature and $\boldsymbol{\mu}$ is the internal state vector.

It is postulated that there exists the free energy function

$$\psi = \hat{\psi}(\mathbf{e}, \mathbf{F}, \vartheta; \boldsymbol{\mu}), \tag{19}$$

and the evolution equation for the internal state vector $\boldsymbol{\mu}$ is assumed in the form

$$L_v\boldsymbol{\mu} = \hat{\mathbf{m}}(\mathfrak{s}). \tag{20}$$

In the theory which we intend to develop the free energy function (19) and the evolution equation (20) will play a fundamental role.

To investigate a notion of objectivity let us consider a superposed rigid body motion given by a map (cf. Truesdell and Noll, 1965)

$$\mathbf{x}^* = \mathbf{c}(t) + \mathbf{Q}(t)\cdot\mathbf{x}$$

or

$$\boldsymbol{\phi}^*(\mathbf{X}, t) = \mathbf{c}(t) + \mathbf{Q}(t)\cdot\boldsymbol{\phi}(\mathbf{X}, t), \tag{21}$$

where $\mathbf{c}(t)$ is a vector function of time and $\mathbf{Q}(t)$ is a time-dependent, proper orthogonal transformation.

The deformation gradient for a new motion is given by

$$\mathbf{F}^* = \frac{\partial}{\partial \mathbf{X}}\boldsymbol{\phi}^* = \mathbf{Q}(t)\cdot\mathbf{F}. \tag{22}$$

A spatial tensor field is said to transform objectively under superposed rigid body motions if it transforms according to the standard rules of tensor analysis.

Similarly we can consider any superposed spatial diffeomorphism $\boldsymbol{\xi}: \mathbb{R}^3 \to \mathbb{R}^3$. This gives

$$\boldsymbol{\phi}^* = \boldsymbol{\xi} \circ \boldsymbol{\phi}: \Gamma \to \mathbb{R}^3 \tag{23}$$

with deformation gradient

$$\mathbf{F}^* = \frac{\partial \boldsymbol{\xi}}{\partial \mathbf{x}}\cdot\mathbf{F}, \tag{24}$$

where $T\boldsymbol{\xi} = \partial\boldsymbol{\xi}/\partial\mathbf{x}$ is the associated relative deformation gradient.

Generally, any spatial tensor field $\mathbf{t}$ is said to transform objectively under superposed diffeomorphism ξ if it transforms according to the rule

$$\mathbf{t}^* = \xi_* \mathbf{t}, \tag{25}$$

where ξ_* is the push-forward operation.

Let $\mathbf{t}$ be a given time-dependent spatial tensor field on $\mathscr{S}$ and let $\mathbf{t}$ transform objectively. Let $\mathbf{v}^*$ be the velocity field of ϕ^*. Then we have (cf. Marsden and Hughes, 1983)

$$L_{\mathbf{v}^*} \mathbf{t}^* = \xi_* (L_{\mathbf{v}} \mathbf{t}). \tag{26}$$

This means that objective tensors have objective Lie derivatives.

There are two levels of objectivity for the constitutive structures:

(i) Frame invariance. The constitutive structure should be invariant with respect to superposed rigid body motions (cf. Truesdell and Noll, 1965).

(ii) Spatial covariance. The constitutive structure should be invariant with respect to diffeomorphisms (cf. Marsden and Hughes, 1983).

Frame invariance leads to objectivity with respect to isometries, while the notion of spatial covariance furnishes the constitutive structure with objectivity property with respect to diffeomorphisms.

It is worthwhile considering two important particular cases (cf. Simo, 1988).

1. Let $\xi : \mathbb{R}^3 \to \mathbb{R}^3$ be an isometry (rigid body motion), then the requirement of spatial covariance reduces to the frame invariance conditions.

2. By choosing $\xi : \mathbb{R}^3 \to \mathbb{R}^3$ as the inverse of actual motion, i.e. $\xi = \phi^{-1}$, we have

$$\xi_* = \phi^*. \tag{27}$$

Then

$$\begin{aligned} \mathbf{e}^* &= \phi^* \mathbf{e} = \mathbf{E}, \\ \mathbf{F}^* &= \mathbf{F}^{-1} \cdot \mathbf{F} = \mathbf{I}, \\ \boldsymbol{\mu}^* &= \phi^* \boldsymbol{\mu} = \mathbf{M}, \end{aligned} \tag{28}$$

where $\mathbf{E}$ is the Lagrangian strain tensor and $\mathbf{M}$ denotes the internal state vector in the material description. Finally (19) reduces to the free energy function

$$\Psi = \hat{\Psi}(\mathbf{E}, \theta; \mathbf{M}) \tag{29}$$

for the material setting. This result has very important consequence for the constitutive modelling.

In further considerations we shall take advantage of both notions of objectivity.

3. THERMOMECHANICAL COUPLINGS FOR J_2-FLOW THEORY

3.1. *Flow rule and isotropic hardening*

Let us introduce the yield criterion in the following form:

$$f - \kappa = 0, \tag{30}$$

where the yield function f is assumed as

$$f = \hat{f}(\tau, \mathbf{g}) = J_2 = \tfrac{1}{2} \tau'^{ab} \tau'^{cd} g_{ac} g_{bd} \tag{31}$$

and the work-hardening–softening function κ is postulated in the form (cf. Nemes *et al.*, 1990)

$$\kappa = \hat{\kappa}(\varepsilon^p, \vartheta) = [\kappa_1 + (\kappa_0 - \kappa_1)\,\mathrm{e}^{-h_1(\vartheta)\varepsilon^p}]^2(1-\omega\vartheta), \tag{32}$$

where κ_0 is a material constant related to the initial yield stress, κ_1 is the saturation hardening stress, $h_1 = h_1(\vartheta)$ is a temperature-dependent hardening function, ε^p denotes the equivalent plastic deformation

$$\varepsilon^p = \int_0^t (\tfrac{2}{3}\mathbf{d}^p : \mathbf{d}^p)^{1/2}\,\mathrm{d}t' \tag{33}$$

and

$$\vartheta = \frac{\vartheta - \vartheta_0}{\vartheta_0}, \quad \omega = \text{const.} \tag{34}$$

The function κ in the form (32) describes the saturation of the hardening of the material as the plastic deformation progresses.

Linear approximation of the function $\mathrm{e}^{-h_1(\vartheta)\varepsilon^p}$ gives

$$\kappa = \kappa_0^2[1 + h(\vartheta)\varepsilon^p]^2(1-\omega\vartheta), \tag{35}$$

where

$$h(\vartheta) = \left(\frac{\kappa_1}{\kappa_0} - 1\right)h_1(\vartheta). \tag{36}$$

The flow rule is postulated in the form

$$\mathbf{d}^p = \Lambda\mathbf{P}, \tag{37}$$

where

$$\mathbf{P} = \frac{1}{2\sqrt{J_2}}\frac{\partial \hat{f}}{\partial \boldsymbol{\tau}} = \left(\frac{\boldsymbol{\tau}'}{2\sqrt{J_2}}\right)^{\flat} \tag{38}$$

and the symbol $\flat$ denotes the index lowering operator.

Fulfilment of the consistency condition $\dot{f} - \dot{\kappa} = 0$ gives

$$\Lambda = \left\langle \frac{1}{H}(\mathbf{P} : \dot{\boldsymbol{\tau}} + \pi\dot{\vartheta}) \right\rangle, \tag{39}$$

where the symbol $\langle(x)\rangle$ defines the ramp function

$$\langle(x)\rangle = \frac{x + |x|}{2}, \tag{40}$$

and is used to express the loading/unloading criterion; the isotropic hardening modulus H is determined by

$$H = \frac{1}{2\sqrt{3J_2}}\frac{\partial \hat{\kappa}}{\partial \varepsilon^p} = \frac{h_1(\vartheta)(\kappa_1 - \kappa_0)}{\sqrt{3J_2}}[\kappa_1 + (\kappa_0 - \kappa_1)\,\mathrm{e}^{-h_1(\vartheta)\varepsilon^p}](1-\omega\vartheta)\,\mathrm{e}^{-h_1(\vartheta)\varepsilon^p}, \tag{41}$$

and

$$\pi = -\frac{1}{2\sqrt{J_2}}\frac{\partial\hat{\kappa}}{\partial\vartheta} = \frac{1}{2\sqrt{J_2}}[\kappa_1+(\kappa_0-\kappa_1)\,e^{-h_1(\vartheta)\varepsilon^p}]\left\{\frac{\omega}{\vartheta_0}[\kappa_1+(\kappa_0-\kappa_1)\,e^{-h_1(\vartheta)\varepsilon^p}]\right.$$
$$\left.+2(\kappa_0-\kappa_1)\frac{dh_1(\vartheta)}{d\vartheta}\varepsilon^p(1-\omega\vartheta)\,e^{-h_1(\vartheta)\varepsilon^p}\right\}. \quad (42)$$

Finally the flow rule takes the following form:

$$\mathbf{d}^p = \left\langle\frac{1}{H}(\mathbf{P}:\dot{\tau}+\pi\dot{\vartheta})\right\rangle\mathbf{P}. \quad (43)$$

It is noteworthy that our consideration is valid for any material function $\kappa = \hat{\kappa}(\varepsilon^p, \vartheta)$. Furthermore, we can also postulate the evolution equation for κ, e.g.

$$\dot{\kappa} = \hat{h}_1(\jmath)(\mathbf{d}^p:\mathbf{d}^p)^{1/2}+\hat{h}_2(\jmath)\dot{\vartheta} \quad (44)$$

then

$$H = \hat{h}_1(\jmath)\frac{1}{2\sqrt{2J_2}} \quad (45)$$

and

$$\pi = -\frac{1}{2\sqrt{J_2}}\hat{h}_2(\jmath). \quad (46)$$

3.2. *Evolution equation for the internal state vector*

It is reasonable to present here a brief discussion of the main features of a rate-independent plastic model of a material. The first important property is connected with permanent deformations. This is the result of different paths assumed for the loading and unloading processes. The unloading process, starting from the achieved elastic–plastic state, follows a path in the stress space different from that of the loading process.

The second feature of the plastic model is its time independence. So, the constitutive equations as well as the evolution equations for an elastic–plastic material have to be invariant under the time scale changes.

Both these features, namely the occurrence of permanent deformations and the time-independent behaviour of a material, are characteristic of inviscid plastic models.

The internal state vector $\boldsymbol{\mu}$ is introduced to describe dissipation effects occurring during the thermoplastic flow process. The evolution equation for the internal state vector $\boldsymbol{\mu}$ postulated in the form (20) has to satisfy these two main properties of a plastic model.

To fulfil this requirement we have to assume

$$L_v\boldsymbol{\mu} = \mathbf{m}(\jmath)\left\langle\frac{1}{H}(\mathbf{P}:\dot{\tau}+\pi\dot{\vartheta})\right\rangle, \quad (47)$$

where the material function $\mathbf{m}(\jmath)$ remains to be determined.

3.3. *Thermodynamic restrictions*

Consider balance principles as follows:

(i) Conservation of mass. Let us assume that $\phi(\mathbf{X}, t)$ is a C^1 regular motion. A function $\rho(\mathbf{x}, t)$ is said to obey conservation of mass if

$$\dot{\rho}+\rho\,\mathrm{div}\,v = 0 \quad (48)$$

or

$$\rho(\mathbf{x}, t)J(\mathbf{X}, t) = \rho_{\text{Ref}}(\mathbf{X}). \tag{49}$$

(ii) Balance of momentum. Assume that conservation of mass and balance of momentum hold. If there is no external body force field, then

$$\rho\dot{\mathbf{v}} = \operatorname{div}\boldsymbol{\sigma}. \tag{50}$$

(iii) Balance of moment of momentum. Let conservation of mass and balance of momentum hold. Then balance of moment of momentum holds if and only if τ is symmetric.

(iv) Balance of energy. Assume the following balance principles hold: conservation of mass, balance of momentum, balance of moment of momentum and balance of energy. If there is no external heat supply then

$$\rho(\dot{\psi} + \vartheta\dot{\eta} + \eta\dot{\vartheta}) + \operatorname{div}\mathbf{q} = \frac{\rho}{\rho_{\text{Ref}}}\tau : \mathbf{d}, \tag{51}$$

where η denotes the specific (per unit mass) entropy and $\mathbf{q}$ is the heat vector field.

(v) Entropy production inequality. Assume conservation of mass, balance of momentum, moment of momentum, energy and the entropy production inequality hold. Then the reduced dissipation inequality is satisfied:

$$\frac{1}{\rho_{\text{Ref}}}\tau : \mathbf{d} - (\eta\dot{\vartheta} + \dot{\psi}) - \frac{1}{\rho\vartheta}\mathbf{q}\cdot\operatorname{grad}\vartheta \geqslant 0. \tag{52}$$

Let us introduce *the axiom of entropy production*: for any regular motion of a body $\mathscr{B}$ the constitutive functions are assumed to satisfy the reduced dissipation inequality (52).

Then the constitutive assumption (19) and the evolution equation for the internal state vector μ (47) together with the reduced dissipation inequality (52) lead to the results as follows

$$\tau = \rho_{\text{Ref}}\frac{\partial\psi}{\partial\mathbf{e}}, \quad \eta = -\frac{\partial\psi}{\partial\vartheta},$$

$$-\frac{\partial\psi}{\partial\mu}\cdot L_v\mu - \frac{1}{\rho\vartheta}\mathbf{q}\cdot\operatorname{grad}\vartheta \geqslant 0. \tag{53a-c}$$

We define the rate of internal dissipation by

$$\vartheta\dot{i} = -\frac{\partial\psi}{\partial\mu}\cdot L_v\mu = -\frac{\partial\psi}{\partial\mu}\cdot\mathbf{m}(\omega)\left\langle\frac{1}{H}(\mathbf{P} : \dot{\tau} + \pi\dot{\vartheta})\right\rangle. \tag{54}$$

Equation (54) expresses a very important feature of thermoplastic response of a material, namely that the rate of internal dissipation occurs only during the loading process.

3.4. *Rate type constitutive relation*

Operating on the stress relation (53a) with the Lie derivative and keeping the history constant (the internal state vector constant), we obtain

$$L_v\tau = \mathscr{L}^e\cdot\mathbf{d}^e - \mathscr{L}^{th}\dot{\vartheta} \tag{55}$$

where

$$\mathscr{L}^e = \rho_{\text{Ref}}\frac{\partial^2\psi}{\partial\mathbf{e}^2}, \quad \mathscr{L}^{th} = -\rho_{\text{Ref}}\frac{\partial^2\psi}{\partial\mathbf{e}\,\partial\vartheta} \tag{56}$$

denote the elastic moduli and the thermal stress coefficients, respectively.

Let us generalize the relation (55) for an elastoplastic flow process. Then taking (10) into account we can write

$$L_{\nu}\tau = \mathscr{L}^{e} \cdot (\mathbf{d} - \mathbf{d}^{p}) - \mathscr{L}^{th}\vartheta. \tag{57}$$

Notice that in view of eqns (13) and (16) the flow rule (43) can be written accordingly in two equivalent forms:

$$\mathbf{d}^{p} = \left\langle \frac{1}{H}[\mathbf{P} : (L_{\nu}\tau + \mathbf{d}\cdot\tau + \tau\cdot\mathbf{d}) + \pi\dot{\vartheta}] \right\rangle P \tag{58}$$

or

$$\mathbf{d}^{p} = \left\langle \frac{1}{H}(\mathbf{P} : \overset{\nabla}{\tau} + \pi\dot{\vartheta}) \right\rangle \mathbf{P}. \tag{59}$$

Substituting (58) into (57) yields the evolution equation for the Kirchhoff stress tensor τ in the following form (cf. Perzyna, 1989, 1990):

$$L_{\nu}\tau = \mathscr{L} \cdot \mathbf{d} - \boldsymbol{\varkappa}\dot{\vartheta}, \tag{60}$$

where

$$\mathscr{L} = \left[\mathbf{I} - \frac{\frac{1}{H}\mathscr{L}^{e}\cdot\mathbf{PP}}{1 + \frac{1}{H}(\mathscr{L}^{e}\cdot\mathbf{P}) : \mathbf{P}}\right] \cdot \left[\mathscr{L}^{e} - \frac{1}{H}\mathscr{L}^{e}\cdot\mathbf{P}(\mathbf{P}\cdot\tau + \tau\cdot\mathbf{P})\right],$$

$$\boldsymbol{\varkappa} = \left[\mathbf{I} - \frac{\frac{1}{H}\mathscr{L}^{e}\cdot\mathbf{PP}}{1 + \frac{1}{H}(\mathscr{L}^{e}\cdot\mathbf{P}) : \mathbf{P}}\right] \cdot \left[\mathscr{L}^{th} + \frac{1}{H}\pi\mathscr{L}^{e}\cdot\mathbf{P}\right]. \tag{61a,b}$$

Substituting the flow rule (59) and the relation (17) into eqn (57) gives the alternative form of the rate type constitutive equation

$$\overset{\nabla}{\tau} = \hat{\mathscr{L}} \cdot \mathbf{d} - \boldsymbol{\varkappa}\dot{\vartheta}, \tag{62}$$

where

$$\hat{\mathscr{L}} = \left[\mathbf{I} - \frac{\frac{1}{H}\mathscr{L}^{e}\cdot\mathbf{PP}}{1 + \frac{1}{H}(\mathscr{L}^{e}\cdot\mathbf{P}) : \mathbf{P}}\right] \cdot [\mathscr{L}^{e} + \mathbf{g}\tau + \tau\mathbf{g}] \tag{63}$$

and $\boldsymbol{\varkappa}$ is determined by eqn (61b).

It is noteworthy that the rate type formulations (60) and (62) are materially isomorphic or equivalent, that is each of them describes the same material.

There is now an extensive technical literature dealing with an elastic–plastic model of solids in which there is no distinction made between the matrix $\mathscr{L}^{e} + \mathbf{g}\tau + \tau\mathbf{g}$ and $\mathscr{L}^{e}$. In other words for practical purposes in such a model the term $g^{ac}\tau^{bd} + g^{ad}\tau^{bc}$ compared to the matrix $(\mathscr{L}^{e})^{abcd}$ is neglected.

The simplified theory obtained in this way is objective only with respect to superposed rigid body motions. Such a theory does not meet the much stronger condition of spatial covariance.

3.5. *Thermomechanical couplings*

Substituting $\dot{\psi}$ into (51) and taking into account the results (53) gives

$$\rho\vartheta\dot{\eta} = -\operatorname{div}\mathbf{q}+\rho\vartheta\hat{\imath}. \tag{64}$$

Operating on the entropy relation (53b) with the Lie derivative and substituting the result together with the assumed Fourier constitutive law for the heat flux in the form

$$\mathbf{q} = -k\,\operatorname{grad}\vartheta, \tag{65}$$

where k is the conductivity coefficient, into (64) we obtain the heat conduction equation as follows:

$$\rho c_p\dot{\vartheta} = \operatorname{div}(k\,\operatorname{grad}\vartheta)+\vartheta\frac{\rho}{\rho_{\mathrm{Ref}}}\frac{\partial\tau}{\partial\vartheta}:\mathbf{d}+\rho\chi\left\langle\frac{1}{H}(\mathbf{P}:\dot{\tau}+\pi\dot{\vartheta})\right\rangle, \tag{66}$$

where

$$\chi = -\left(\frac{\partial\psi}{\partial\mu}-\vartheta\frac{\partial^2\psi}{\partial\vartheta\,\partial\mu}\right)\cdot\mathbf{m}(\jmath), \tag{67}$$

and c_p denotes the specific heat and is determined by

$$c_p = -\vartheta\frac{\partial^2\psi}{\partial\vartheta^2}. \tag{68}$$

The main problem remaining to be solved is connected with the determination of the constitutive function $\mathbf{m}(\jmath)$. This function plays the crucial role in the description of the rate of internal dissipation (54) as well as of the main contribution to thermomechanical coupling phenomena [cf. eqns (66) and (67)].

To find the answer to this problem let us perform a Legendre transformation.† We define the complementary free energy by

$$\varphi = \hat{\varphi}(\tau,\mathbf{F},\vartheta;\mu) = \frac{1}{\rho_{\mathrm{Ref}}}\tau:\mathbf{e}-\psi(\mathbf{e},\mathbf{F},\vartheta;\mu). \tag{69}$$

Differentiating (69) in τ, we have

$$\mathbf{e} = \rho_{\mathrm{Ref}}\frac{\partial\hat{\varphi}}{\partial\tau}. \tag{70}$$

Operating on the last result with the Lie derivative gives

$$\mathbf{d} = \mathbf{c}\cdot L_{,\tau}+\mathbf{r}\dot{\vartheta}+\rho_{\mathrm{Ref}}\frac{\partial^2\hat{\varphi}}{\partial\tau\,\partial\mu}\cdot L_{,\mu}. \tag{71}$$

† For the application of a Legendre transformation to analysis of elastoplastic properties of material, see Hill and Rice (1973).

where

$$\mathbf{c} = \rho_{\mathrm{Ref}} \frac{\partial^2 \hat{\varphi}}{\partial \tau^2}, \quad \mathbf{r} = \rho_{\mathrm{Ref}} \frac{\partial^2 \hat{\varphi}}{\partial \tau \, \partial \vartheta} \tag{72}$$

define the material compliance tensors.

Identifying the last term in eqn (71) with the rate of plastic deformation yields the identity†

$$\rho_{\mathrm{Ref}} \frac{\partial^2 \hat{\varphi}}{\partial \tau \, \partial \mu} \cdot \mathbf{m}(\jmath) \left\langle \frac{1}{H} (\mathbf{P} : \dot{\tau} + \pi \dot{\vartheta}) \right\rangle = \left\langle \frac{1}{H} (\mathbf{P} : \dot{\tau} + \pi \dot{\vartheta}) \right\rangle \mathbf{P}, \tag{73}$$

so we get the result

$$\mathbf{m}(\jmath) = \frac{1}{\rho_{\mathrm{Ref}}} \left[\frac{\partial^2 \hat{\varphi}}{\partial \tau \, \partial \mu} \right]^{-1} : \mathbf{P}. \tag{74}$$

The answer to the problem of determination of the material function $\mathbf{m}(\jmath)$ can be found in a different way, namely by using the stress relation (53) and operating on it directly with the Lie derivative.

Making use of the definition (67) and taking advantage of the result (74) we get

$$\chi = -\frac{1}{\rho_{\mathrm{Ref}}} \left(\frac{\partial \psi}{\partial \mu} - \vartheta \frac{\partial^2 \psi}{\partial \vartheta \, \partial \mu} \right) \cdot \left[\frac{\partial^2 \hat{\varphi}}{\partial \tau \, \partial \mu} \right]^{-1} : \mathbf{P}. \tag{75}$$

Substituting the result (75) into the heat conduction equation (66) we obtain

$$\rho c_p \dot{\vartheta} = \operatorname{div} (k \operatorname{grad} \vartheta) + \vartheta \frac{\rho}{\rho_{\mathrm{Ref}}} \frac{\partial \tau}{\partial \vartheta} : \mathbf{d} + \zeta \tau : \mathbf{d}^p + \frac{\rho}{\rho_{\mathrm{Ref}}} \vartheta \frac{\partial^2 \psi}{\partial \vartheta \, \partial \mu} \cdot \left[\frac{\partial^2 \hat{\varphi}}{\partial \tau \, \partial \mu} \right]^{-1} : \mathbf{d}^p, \tag{76}$$

where the new notation

$$\zeta \tau = -\frac{\rho}{\rho_{\mathrm{Ref}}} \frac{\partial \psi}{\partial \mu} \cdot \left[\frac{\partial^2 \hat{\varphi}}{\partial \tau \, \partial \mu} \right]^{-1} \tag{77}$$

is introduced.

To determine ζ we have the identity

$$\zeta \rho_{\mathrm{Ref}} \frac{\partial \psi}{\partial \mathbf{e}} = -\frac{\rho}{\rho_{\mathrm{Ref}}} \frac{\partial \psi}{\partial \mu} \cdot \left[\frac{\partial^2 \hat{\varphi}}{\partial \tau \, \partial \mu} \right]^{-1}. \tag{78}$$

4. ADIABATIC PROCESS

4.1. *Fundamental equation for the adiabatic process*

The thermodynamic process is assumed to be adiabatic, i.e.

$$\mathbf{q} = 0. \tag{79}$$

This assumption is satisfied for the thermoplastic flow process before localization takes place, then the distribution of plastic deformation as well as the rate of plastic deformation is homogeneous.

† Similar identification of the rate of plastic deformation was first peformed by Rice (1971); cf. also Hill and Rice (1973).

The term div (k grad ϑ) in the heat conduction equation (66) vanishes. Then to describe thermomechanical constitutive properties of a material we have two coupled evolution equations, namely for the Kirchhoff stress tensor (60) and for temperature

$$c_p\dot{\vartheta} = \vartheta\frac{1}{\rho_{\mathrm{Ref}}}\frac{\partial\tau}{\partial\vartheta}:\mathbf{d}+\chi\left\langle\frac{1}{H}(\mathbf{P}:\dot{\tau}+\pi\dot{\vartheta})\right\rangle. \tag{80}$$

Equation (80) can be written in the form

$$\dot{\vartheta} = \frac{\vartheta H}{\rho_{\mathrm{Ref}}(c_pH-\chi\pi)}\frac{\partial\tau}{\partial\vartheta}:\mathbf{d}+\frac{\chi}{c_pH-\chi\pi}\mathbf{P}:\dot{\tau}. \tag{81}$$

Taking advantage of eqn (13) in (81) and substituting the result into eqn (60) gives

$$L_v\tau = \mathbb{L}\cdot\mathbf{d}, \tag{82}$$

where

$$\mathbb{L} = \left[\mathbf{I}+\frac{\chi}{c_pH-\chi\pi}\varkappa\mathbf{P}\right]^{-1}\cdot\left\{\mathscr{L}-\frac{1}{c_pH-\chi\pi}\left[\frac{\vartheta H}{\rho_{\mathrm{Ref}}}\varkappa\frac{\partial\tau}{\partial\vartheta}2\chi\varkappa(\mathbf{P}\cdot\tau+\tau\cdot\mathbf{P})\right]\right\}. \tag{83}$$

The result (82) is of great importance to constitutive modelling of thermomechanical coupling phenomena in adiabatic processes and to the investigation of the conditions for the localization of plastic deformation.

It is noteworthy that the fundamental matrix $\mathbb{L}$ [cf. eqn (83)] in the evolution equation (82) describes all the introduced thermomechanical coupling effects.

Proceeding similarly but replacing the Lie derivative equation for the Kirchhoff stress tensor (60) by the Zaremba–Jaumann rate equation (62) we get

$$\overset{\nabla}{\tau} = \hat{\mathbb{L}}\cdot\mathbf{d}, \tag{84}$$

where

$$\mathbb{L} = \left[\mathbf{I}+\frac{\chi}{c_pH-\chi\pi}\varkappa\mathbf{P}\right]^{-1}\cdot\left[\hat{\mathscr{L}}-\frac{H\vartheta}{\rho_{\mathrm{Ref}}(c_pH-\chi\pi)}\varkappa\frac{\partial\tau}{\partial\vartheta}\right]. \tag{85}$$

It is worthwhile pointing out once more that the evolution equation (84) is materially isomorphic with (82).

4.2. *Main contribution to thermomechanical coupling*

Let us consider the heat conduction equation (76). For an adiabatic process this equation takes the form

$$\rho c_p\dot{\vartheta} = \vartheta\frac{\rho}{\rho_{\mathrm{Ref}}}\frac{\partial\tau}{\partial\vartheta}:\mathbf{d}+\zeta\tau:\mathbf{d}^p+\frac{\rho}{\rho_{\mathrm{Ref}}}\vartheta\frac{\partial^2\psi}{\partial\vartheta\,\partial\mu}\cdot\left[\frac{\partial^2\dot{\varphi}}{\partial\tau\,\partial\mu}\right]^{-1}:\mathbf{d}^p. \tag{86}$$

The second term on the right-hand side represents the main contribution to the thermomechanical coupling phenomena. It generates the internal heating caused by the rate of internal dissipation during the adiabatic process considered.

The first and third terms represent the cross coupling effects, the first is caused by the dependence of the stress tensor on temperature while the third is induced by the same dependence of the generalized force conjugates to the internal state vector μ.

The cross coupling effects influence the evolution of temperature [cf. eqn (86)] through the second order terms when compared with the internal dissipation term. Their contribution to internal heating during the adiabatic process considered is small.

This suggests that these two terms can be neglected in some considerations like the investigation of the conditions of the localization of plastic deformation along the shear band.

So, it is reasonable to consider the evolution equation for temperature in the form

$$\rho c_p \dot{\vartheta} = \zeta \boldsymbol{\tau} : \mathbf{d}^p, \tag{87}$$

where ζ is determined by (78).

Let us consider a set of two coupled evolution equations, for the stress tensor (60) and for temperature (87). This set is reduced again to the fundamental equation of the form

$$L_v \boldsymbol{\tau} = \tilde{\mathbb{L}} \cdot d, \tag{88}$$

where

$$\tilde{\mathbb{L}} = \left[\mathbf{I} + \frac{\zeta \sqrt{J_2}}{H\rho c_p - \zeta \pi \sqrt{J_2}} \varkappa \mathbf{P} \right]^{-1} \cdot \left\{ \mathscr{L} - \frac{2\zeta \sqrt{J_2}}{H\rho c_p - \zeta \pi \sqrt{J_2}} \varkappa [\mathbf{P} \cdot \boldsymbol{\tau} + \boldsymbol{\tau} \cdot \mathbf{P}] \right\}. \tag{89}$$

Let us consider now a similar set of two coupled evolution equations for the stress tensor (62) and for temperature (87). This gives

$$\overset{\nabla}{\boldsymbol{\tau}} = \hat{\mathbb{L}} \cdot \mathbf{d} \tag{90}$$

where

$$\hat{\mathbb{L}} = \left[\mathbf{I} + \frac{\zeta \sqrt{J_2}}{H\rho c_p - \zeta \pi \sqrt{J_2}} \varkappa \mathbf{P} \right]^{-1} \cdot \mathscr{L}. \tag{91}$$

5. CONDITIONS FOR LOCALIZATION ALONG SHEAR BAND

5.1. *Criteria for localization*

It is noteworthy that the important result obtained for an adiabatic process in the form of the evolution equation for the Kirchhoff stress tensor [cf. eqns (82), (84), (88) and (90)] allows us to use the standard bifurcation method in examination of the shear band localization condition in elastic–plastic solids when thermomechanical couplings are taken into consideration.

This method leads to the criterion for a stationary discontinuity that can be predicted in terms of the constitutive relations of the material under the condition of pre-localization.

The fundamental principles of this method were given by Hadamard (1903) for elastic solids and extended to elastic–plastic solids by Thomas (1961), Hill (1962) and Mandel (1966).

The theory of the localization of plastic deformation along shear band was developed mainly by Rice (1973, 1976), Rudnicki and Rice (1975), Needleman and Rice (1978), Rice and Rudnicki (1980) and Lippmann (1986).

In the previous papers of the authors (Duszek and Perzyna, 1988a,b) the standard bifurcation method was used to investigate the shear band localization conditions for finite elastic–plastic rate-independent deformations of damaged solids.

The main objective of this section is to investigate the influence of thermomechanical coupling effects on shear band localization criteria.

The procedure of analysis developed by Rice (1976), Rudnicki and Rice (1975) and Rice and Rudnicki (1980) is adopted to investigate the localization conditions in an adiabatic process.

5.1.1. *The Lie derivative formulation.* Let $\{X^I\}$ and $\{x^i\}$ denote Cartesian coordinate systems on $\mathscr{B}$ and $\mathscr{S}$, respectively. Consider a homogeneous solid body $\mathscr{B}$ subjected to a thermodynamic process of elasto–plastic flow phenomena.

The initial-boundary value problem consists of finding ϕ and ϑ as functions of $\mathbf{x}$ and t such that the field equations:

$$\begin{aligned}
&\rho\dot{\mathbf{v}} + \operatorname{div}\boldsymbol{\sigma} = 0,\\
&L_v\boldsymbol{\tau} = \mathscr{L}\cdot\mathbf{d} - \boldsymbol{\varkappa}\dot{\vartheta},\\
&L_v\mu = \mathbf{m}(\cdot)\left\langle \frac{1}{H}(\mathbf{P}:\dot{\boldsymbol{\tau}} + \pi\dot{\vartheta})\right\rangle,\\
&\rho c_p\dot{\vartheta} = \operatorname{div}(k\operatorname{grad}\vartheta) + \vartheta\frac{\rho}{\rho_{\mathrm{Ref}}}\frac{\partial\boldsymbol{\tau}}{\partial\vartheta}:\mathbf{d} + \rho\chi\left\langle\frac{1}{H}(\mathbf{P}:\dot{\boldsymbol{\tau}} + \pi\dot{\vartheta})\right\rangle,
\end{aligned} \tag{92a–d}$$

the boundary conditions:

$$\begin{aligned}
&\text{tractions } (\boldsymbol{\tau}\cdot\mathbf{n})^{\cdot} \text{ are prescribed on } \partial\mathscr{B},\\
&\text{temperature } \vartheta \text{ is prescribed on } \partial\mathscr{B},
\end{aligned}$$

and initial conditions:

$$\phi,\ \mathbf{v},\ \mu \text{ and } \vartheta \quad \text{are given at} \quad t = 0,$$

are satisfied.

In many practical situations the problem considered can be treated as quasi-static and adiabatic. Then based on the results of Section 4 the field equations (92) can be reduced to

$$\begin{aligned}
&\operatorname{div}\boldsymbol{\sigma} = 0,\\
&L_v\boldsymbol{\tau} = \mathbb{L}\cdot\mathbf{d},
\end{aligned} \tag{93a,b}$$

with prescribed tractions $(\boldsymbol{\tau}\cdot\mathbf{n})^{\cdot}$ on $\partial\mathscr{B}$.

Let us assume that the boundary conditions are such that a body sustains a uniform stress $\boldsymbol{\tau}^0$ and uniform temperature ϑ^0. The response to a homogeneous velocity gradient field $(\partial\mathbf{v}/\partial\mathbf{x})^0$ is the homogeneous stress rate $\dot{\boldsymbol{\tau}}^0$ which satisfies the quasi-static field equations (93).

Conditions are sought for which the state of the solid body $\mathscr{B}$ allows the field equations to be satisfied for an alternate field

$$\frac{\partial\mathbf{v}}{\partial\mathbf{x}} = \left(\frac{\partial\mathbf{v}}{\partial\mathbf{x}}\right)^0 + \Delta\left(\frac{\partial\mathbf{v}}{\partial\mathbf{x}}\right) \tag{94}$$

in which the jump $\Delta(\partial\mathbf{v}/\partial\mathbf{x})$ is a function only of distance across a planar band and vanishes outside the band. If the velocity is to be continuous at bifurcation, the compatibility condition

$$\Delta\left(\frac{\partial \mathbf{v}}{\partial \mathbf{x}}\right) = \mathbf{kn} \tag{95}$$

must be satisfied, where $\mathbf{n}$ denotes the unit normal to the plane of the band and the magnitude of jump $\mathbf{k}$ is a function of distance across the band $(\mathbf{n}\cdot\mathbf{x})$ only, and is zero outside.

Equilibrium must also be satisfied at the inception of bifurcation. This is expressed in rate form as

$$\frac{\partial}{\partial x^i}\left(\frac{\partial \sigma^{ij}}{\partial t}\right) = \frac{\partial}{\partial x^i}\left(\dot{\tau}^{ij} - \tau^{ij} d_{kk} - v_k \frac{\partial \tau^{ij}}{\partial x^k}\right) = 0. \tag{96}$$

Because bifurcation is from the homogeneous state $\tau = \tau^0$ the rate equilibrium condition reduces at the instant considered to

$$\frac{\partial \dot{\tau}^{ij}}{\partial x^i} - \tau^{ij}\frac{\partial d_{kk}}{\partial x^i} = 0. \tag{97}$$

Equations (95) and (97) require that $\mathbf{n}\cdot\dot{\tau}$ has the same value inside and outside of the band, i.e.

$$\mathbf{n}\cdot\Delta\dot{\tau} - (\mathbf{n}\cdot\tau)(\mathbf{k}\cdot\mathbf{n}) = 0, \tag{98}$$

where $\Delta\dot{\tau} = \dot{\tau} - \dot{\tau}^0$.

Taking advantage of the rate type constitutive relation (93b), the homogeneous field outside the band has to satisfy

$$L_v\tau^0 = \mathbb{L}^0\cdot\mathbf{d}^0, \tag{99}$$

and inside the band the corresponding equation

$$L_v\tau = \mathbb{L}\cdot\mathbf{d}. \tag{100}$$

The compatibility condition (95) can be expressed in terms of $\mathbf{d}$ by

$$\Delta\mathbf{d} = \mathbf{d} - \mathbf{d}^0 = \tfrac{1}{2}(\mathbf{kn} + \mathbf{nk}). \tag{101}$$

Assuming that the constitutive response remains continuous at the inception of localization, i.e.

$$\Delta\mathbb{L} = \mathbb{L} - \mathbb{L}^0 = 0, \tag{102}$$

then eqns (97)–(102) yield

$$(n_i\mathbb{L}^{ijkl}n_l + n_i\tau^{ij}n_l\delta^{lk})k_k = 0. \tag{103}$$

The last two terms in parentheses arise due to the difference between $L_v\tau$ and $\dot{\tau}$.

The onset of the localization occurs at the first instant in the deformation history for which a non-trivial solution of eqn (103) exists. Thus, the necessary condition for a localized shear band to be formed is

$$\det [n_i \mathbb{L}^{ijkl} n_l + n_i \tau^{il} n_l \delta^{jk}] = 0. \tag{104}$$

For simplicity let us introduce rectangular Cartesian coordinates $\{x^i\}$ in such a way that $\mathbf{n}$ is in the x_2-direction. Then eqn (104) takes the following form:

$$\det [\mathbb{L}^{2jk2} + \tau^{22} \delta^{jk}] = 0. \tag{105}$$

5.1.2. *The Zaremba–Jaumann rate formulation.* Replacing the rate constitutive equation (93b) by (84) and proceeding as for the Lie derivative we get the necessary condition for the localization along the shear band in the form

$$\det [n_i \hat{\mathbb{L}}^{ijkl} n_l + \tfrac{1}{2}(n_i \tau^{il} n_l \delta^{jk} - n_i \tau^{ij} n_l \delta^{lk} - n_l n_i \tau^{ik} \delta^{lj} - \tau^{ik})] = 0. \tag{106}$$

Assuming for simplicity that $\mathbf{n}$ is in the x_2-direction, we have

$$\det [\hat{\mathbb{L}}^{2jk2} + \tfrac{1}{2}(\tau^{22} \delta^{jk} - \tau^{2j} \delta^{2k} - \tau^{2k} \delta^{2j} - \tau^{jk})] = 0. \tag{107}$$

5.2. *Simplifications*

To make possible analytical investigation of criteria for localization we introduce some simplifications.

(i) Assume that the Lie derivative $L_v \tau$ is approximated by the material derivative $\dot{\tau}$, i.e. $L_v \tau \approx \dot{\tau}$ (or $\overset{\nabla}{\tau} \approx \dot{\tau}$) in the constitutive laws formulated.

(ii) The evolution equation for temperature (92d) is replaced by (87). That means that only the main contribution to thermomechanical coupling is taken into consideration (cf. discussion in Section 4.2).

(iii) By analogy with the infinitesimal theory of elasticity we postulate

$$(\bar{\mathcal{L}}^{ve})_{ijkl} = \frac{1}{4G}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \left(\frac{1}{9K} - \frac{1}{6G}\right)\delta_{ij}\delta_{kl} + \tau j^l \delta^{ik}, \tag{108}$$

where G and K denote the shear and bulk moduli, respectively.

(iv) Assume that

$$\bar{\mathcal{L}}^{ve} \cdot \mathcal{L}^{th} = \theta \mathbf{I}, \tag{109}$$

where θ is the thermal expansion in the elastic range.

Let us first discuss superposition of (i) and (ii) into the constitutive laws. We observe that for this case, eqns (88) and (90) are equivalent and lead to the simplified constitutive law (of course not objective)

$$\dot{\tau} = \mathbb{L} \cdot \mathbf{d}, \tag{110}$$

where

$$\mathbb{L} = \bar{\mathcal{L}} + \mathcal{J} \tag{111}$$

and

$$\bar{\mathscr{P}} = \left[\mathbf{I} - \frac{\dfrac{1}{H}\mathscr{L}^e \cdot \mathbf{PP}}{1 + \dfrac{1}{H}(\mathscr{L}^e \cdot \mathbf{P}) : \mathbf{P}} \right] \cdot \mathscr{L}^e,$$

$$\bar{\mathscr{T}} = -\left[1 + \frac{\zeta\sqrt{J_2}}{H\rho c_p - \zeta\pi\sqrt{J_2}} \boldsymbol{\varkappa} : \mathbf{P} \right]^{-1} \frac{\zeta\sqrt{J_2}}{H\rho c_p - \zeta\pi\sqrt{J_2}} \boldsymbol{\varkappa}\mathbf{P} \cdot \bar{\mathscr{P}}. \tag{112}$$

Superposing the simplifications (iii) and (iv) and neglecting terms of the order of magnitude $\sqrt{J_2}$, we have

$$\bar{\mathscr{P}}^{ijkl} = G(\delta^{ki}\delta^{lj} + \delta^{kj}\delta^{li}) + (K - \tfrac{2}{3}G)\delta^{ij}\delta^{kl} - \frac{1}{H+G}\frac{G}{\sqrt{J_2}}\tau'^{ij}\frac{G}{\sqrt{J_2}}\tau'^{kl},$$

$$\bar{\mathscr{T}}^{ijkl} = -\frac{G^2}{J_2}\frac{G\pi}{(H+G)(H+G-G\Pi)}\tau'^{ij}\tau'^{kl} - \frac{2G^2\Xi}{(H+G-G\Pi)\sqrt{J_2}}\delta^{ij}\tau'^{kl},$$

$$\mathbb{L}^{ijkl} = G(\delta^{ik}\delta^{jl} + \delta^{kj}\delta^{li}) + (K - \tfrac{2}{3}G)\delta^{ij}\delta^{kl} - \frac{G^2}{J_2(H+G-G\Pi)}\tau'^{ij}\tau'^{kl} - \frac{2G^2\Xi}{(H+G-G\Pi)\sqrt{J_2}}\delta^{ij}\tau'^{kl}, \tag{113a-c}$$

where new notations

$$\Pi = \frac{\zeta\pi\sqrt{J_2}}{\rho c_p G} = -\frac{\zeta\dfrac{\partial\hat{\kappa}}{\partial\vartheta}}{2G\rho c_p}, \quad \Xi = \frac{30K\zeta\sqrt{J_2}}{2G\rho c_p}, \tag{114a,b}$$

are introduced.

The necessary condition for localization along shear band now takes the form

$$\det[\mathbb{L}^{2jk2}] = 0. \tag{115}$$

This condition gives

$$(K + \tfrac{4}{3}G)(H+G-G\Pi)J_2 = G^2(\tau'_{22} + 2\sqrt{J_2}\Xi)\tau'_{22} + G(K + \tfrac{4}{3}G)(\tau_{12}^2 + \tau_{23}^2), \tag{116}$$

or

$$H = -G. \tag{117}$$

The solution $H = -G$ is independent of the orientation of the plane within which the shear band localization first takes place. On the other hand the solution (116) does depend very much on this orientation.

To investigate the solution (116) let us assume $n_{\text{II}} = 0$, and require H to be maximum with respect to n_{I}, i.e.

$$\frac{\partial H}{\partial n_{\text{I}}} = 0. \tag{118}$$

After taking advantage of the relations

$$\tau'_{22} = n_{\text{I}}^2(\tau'_{\text{I}} - \tau'_{\text{III}}) + \tau'_{\text{III}},$$
$$\tau_{12}^2 + \tau_{23}^2 = n_{\text{I}}^2(1 - n_{\text{I}}^2)(\tau'_{\text{I}} - \tau'_{\text{III}})^2, \tag{119}$$

the condition (118) yields the result

$$n_{\mathrm{I}}^2 = \frac{(1-\nu)(T_{\max}-T_{\min})+(1-2\nu)(T_{\min}-\Xi)}{T_{\max}-T_{\min}}, \tag{120}$$

where

$$T_{\max} = \frac{\tau'_{\mathrm{I}}}{\sqrt{J_2}}, \quad T = \frac{\tau'_{\mathrm{II}}}{\sqrt{J_2}}, \quad T_{\min} = \frac{\tau'_{\mathrm{III}}}{\sqrt{J_2}}, \tag{121}$$

and

$$\nu = \frac{3K-2G}{2(3K+G)} \tag{122}$$

is Poisson's ratio.

Since $n_{\mathrm{III}}^2 = 1-n_{\mathrm{I}}^2$, we have

$$\tan\beta = \frac{n_{\mathrm{I}}}{n_{\mathrm{III}}} = \left(-\frac{(\nu T+T_{\max})+(1-2\nu)\Xi}{(\nu T+T_{\min})+(1-2\nu)\Xi}\right)^{1/2}, \tag{123}$$

where β denotes the angle between the vector $\mathbf{n}$ and the τ_{III} direction.

Making use of (120) in (116) we obtain

$$\frac{H_{\mathrm{cr}}}{G} = -\frac{1+\nu}{2}\left(T+\frac{1-2\nu}{1+\nu}\Xi\right)^2 + \frac{(1-2\nu)^2}{1-\nu^2}\Xi^2 + \Pi. \tag{124}$$

For

$$T = -\frac{1-2\nu}{1+\nu}\Xi \tag{125}$$

the critical hardening rate takes a maximum value

$$\left(\frac{H_{\mathrm{cr}}}{G}\right)_{\max} = \frac{(1-2\nu)^2}{1-\nu^2}\Xi^2 + \Pi. \tag{126}$$

5.3. *Spatial covariance effects on localization (isothermal processes)*

The main objective at this point is to investigate the influence of the spatial covariance effects on localization criteria. To do this we remove the simplification (i), replace (ii) by the stronger requirement

$$\vartheta = \text{const}, \tag{127}$$

i.e. the process considered is assumed to be isothermal, and takes advantage of (iii).

Under these conditions the rate equation (60) takes the form

$$L_{\nu}\tau = \mathscr{L}\cdot\mathbf{d} \tag{128}$$

where

$$(\mathscr{L})^{ijkl} = G(\delta^{ik}\delta^{jl}+\delta^{il}\delta^{jk})+(K-\tfrac{2}{3}G)\delta^{ij}\delta^{kl}+\tau j^{l}\delta^{ik} - \frac{G^2}{(H+G)J_2}\tau'^{ij}\tau'^{kl} - \frac{G}{(H+G)J_2}\tau'^{ij}\tau'^{kr}\tau^{ls}\delta_{rs}. \tag{129}$$

It is noteworthy that the last term in (129) represents the spatial covariant effects in the constitutive matrix $\mathscr{L}$. So, it would be convenient to write the matrix $\mathscr{L}$ as a sum

$$\mathscr{L} = \bar{\mathscr{L}} + \mathscr{C} \tag{130}$$

where

$$\bar{\mathscr{L}}^{ijkl} = G(\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}) + (K - \tfrac{2}{3}G)\delta^{ij}\delta^{kl} + \tau'^{jl}\delta^{ik} - \frac{G^2}{(H+G)J_2}\tau'^{ij}\tau'^{kl}, \tag{131}$$

cf. with eqn (113a), and

$$\mathscr{C}^{ijkl} = -\frac{G}{(H+G)J_2}\tau'^{ij}\tau'^{kr}\tau'^{ls}\delta_{rs}. \tag{132}$$

The necessary condition for localization (104) now takes the form

$$\det[n_i(\bar{\mathscr{L}}^{ijkl} + \mathscr{C}^{ijkl})n_l + n_i\tau'^{jl}n_l\delta^{ik}] = 0. \tag{133}$$

This condition can be written as

$$\det[n_i(\bar{\mathscr{L}}^{ijkl} + \mathscr{K}^{ijkl})n_l] = 0, \tag{134}$$

where the matrix

$$\mathscr{K}^{ijkl} = \mathscr{C}^{ijkl} + \tau'^{jl}\delta^{ik} \tag{135}$$

describes the influence of the spatial covariance effects on the criterion for localization.

The condition (134) gives the value of the rate hardening modulus H in the form

$$H = H_0 + H_1, \tag{136}$$

where

$$H_0 = \frac{G}{J_2}\left[\frac{1-2\nu}{2(1-\nu)}\tau'^2_{22} + \tau^2_{12} - J_2\right] \tag{137}$$

is the value of the rate hardening modulus computed by neglecting the influence of the spatial covariance terms and the higher order terms (assuming $\mathscr{K} = 0$), and

$$H_1 = \frac{1}{J_2}\left[\frac{\nu(1-2\nu)}{2(1-\nu)^2}\tau'^2_{22}\tau_{22} + \tau^2_{12}\tau'_{11}\right] + O\left(\frac{J_2}{G}\right) \tag{138}$$

determines the contribution of the spatial covariance terms and the higher order terms to the value of the rate hardening modulus H provided the neglected terms are of the order of magnitude J_2/G and smaller (it means that we neglect terms of the order $\sqrt{J_2}/G$ in comparison to 1).

Taking into account the orientation of the plane within which the shear band localization first occurs we obtain the final result in the following form:

$$\frac{H_{cr}}{G} = -\frac{1+\nu}{2}T^2 + \left\{\frac{(\nu-1)(2-\nu)}{2}T^3 + \left[\frac{\nu(1-2\nu)}{6}\frac{J_1}{\sqrt{J_2}} + (1-\nu+\nu^2)T_{\min}\right]T^2 + (1-\nu)T - T_{\min}\right\}\frac{\sqrt{J_2}}{G} + O\left(\frac{J_2}{G^2}\right). \tag{139}$$

The value in brackets { } represents again the main contribution to the value of H implied by the spatial covariance terms. It is reasonable to comment how this main contribution to the value of H has been obtained.

In order to estimate the change in orientation of the plane of localization (and its influence on the critical hardening rate) when spatial covariance effects are taken into consideration, a linear expansion of the function $\partial H/\partial\beta$ about β_0 is used, where β_0 denotes the angle between the vector $\mathbf{n}$ and the τ_{III} direction when the thermal effects as well as the spatial covariance effects are neglected. Denoting by a prime the derivative with respect to β, we get

$$\begin{aligned} H'(\beta) &= H'(\beta_0)+(\beta-\beta_0)H''(\beta_0) \\ &= H'_0(\beta_0)+H'_1(\beta_0)+(\beta-\beta_0)[H''_0(\beta_0)+H''_1(\beta_0)]. \end{aligned} \tag{140}$$

Taking advantage of the conditions

$$H'(\beta)=0 \quad \text{and} \quad H'_0(\beta_0)=0 \tag{141}$$

we obtain

$$\beta-\beta_0 = -\frac{H'_1(\beta_0)}{H''_0(\beta_0)+H''_1(\beta_0)}. \tag{142}$$

The critical hardening rate can be approximated by the linear expansion of the function H about β_0, and making use of eqn (142) and the condition $H'_0(\beta_0)=0$, we have

$$\begin{aligned} H(\beta) &= H_0(\beta_0)+H_1(\beta_0)+(\beta-\beta_0)[H'_0(\beta_0)+H'_1(\beta_0)] \\ &= H_0(\beta_0)+H_1(\beta_0)-\frac{H'^2_1(\beta_0)}{H''_0(\beta_0)+H''_1(\beta_0)} \\ &= H_0(\beta_0)+H_1(\beta_0)+O\left(\frac{J_2}{G}\right), \end{aligned} \tag{143}$$

as has been utilized in (139).

6. DISCUSSION OF THE RESULTS

6.1. *Influence of thermomechanical couplings*

The result (124) for the critical hardening modulus rate H_{cr}/G as a function of the state of stress T may be represented by a family of parabolas, as has been plotted in Fig. 1.

The expression (124) for $\Pi=\Xi=0$ gives the expected result for associated J_2-flow theory, i.e. $(H_{cr}/G)_{max}$ is zero and localization occurs on the plane whose direction is determined by angle

$$\beta = \arctan\left(-\frac{\nu T+T_{max}}{\nu T+T_{min}}\right),$$

cf. Rudnicki and Rice (1975). For this case (i.e. when $\Pi=\Xi=0$) H_{cr} is non-positive, cf. the parabola plotted in Fig. 1 by a broken line.

However, if thermomechanical coupling effects are taken into consideration, the value of the hardening modulus rate for localization can, in general, be positive, cf. the parabola plotted in Fig. 1 by a solid line.

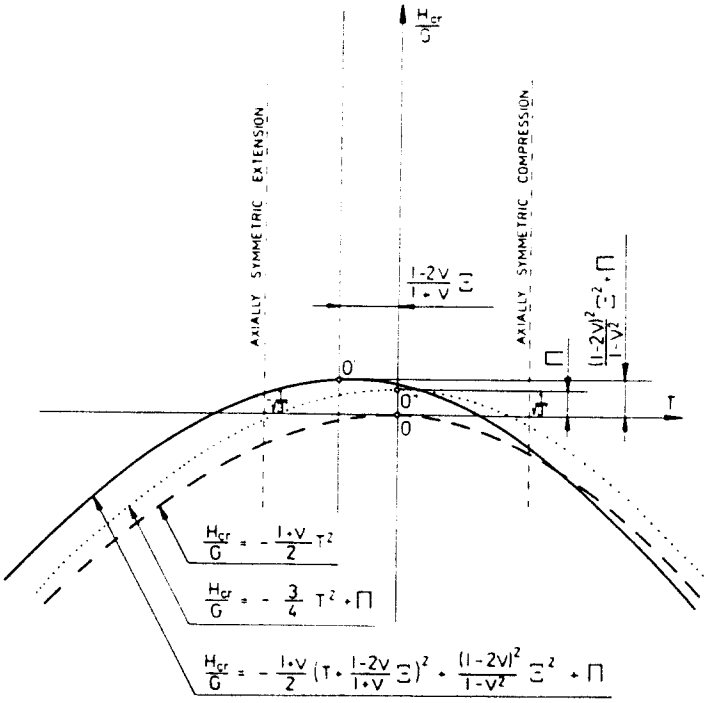

Fig. 1. The variation of the critical hardening modulus rate H_{cr}/G as a function of the state of stress T. (——) Thermoelastic, plastic solid, $\Pi \neq 0$, $\Xi \neq 0$; (- - - -) elastic incompressibility, $\nu = \frac{1}{2}$; (— · — · —) associated J_2-flow theory, $\Pi = 0$, $\Xi = 0$; cf. Rudnicki and Rice (1975).

For given $\Pi \neq 0$ and $\Xi \neq 0$ the parabola described by eqn (124), when compared with the parabola for $\Pi = \Xi = 0$, is shifted

$$\text{in } T\text{-axis by } -\frac{1-2\nu}{1+\nu}\Xi \quad \text{and in} \quad \frac{H_{cr}}{G}\text{-axis by} \quad \frac{(1-2\nu)^2}{1-\nu^2}\Xi^2+\Pi.$$

We consider two important thermal effects, namely thermal expansion, represented by Ξ and thermal plastic softening, represented by Π. Thermal expansion implies that the inclination to instability for axially symmetric compression is different from that for axially symmetric tension. For tension the material is more sensitive to localization than for compression (cf. Fig. 1). Thermal plastic softening causes the material to be more inclined to instability, independently of the state of stress.

For elastic incompressibility, i.e. for $\nu = \frac{1}{2}$, the result (124) takes the form

$$\frac{H_{cr}}{G} = -\tfrac{3}{4}T^2+\Pi \tag{144}$$

(cf. the parabola plotted in Fig. 1 by dotted line), then for $T = 0$

$$\left(\frac{H_{cr}}{G}\right)_{max} = \Pi, \tag{145}$$

and the maximum value of the critical hardening modulus rate for an adiabatic process can again be positive, i.e. $\Pi > 0$. Indeed, since temperature is increasing due to internal dissipation rate, the yield stress $\kappa_0(1-\omega\vartheta)^{1/2}$ as well as the saturation hardening stress $\kappa_1(1-\omega\vartheta)^{1/2}$, are decreasing, similarly $h_1 = h_1(\vartheta)$ is a decreasing function of temperature, i.e. $dh_1/d\vartheta < 0$, then from eqns (42) and (114a) we have $\Pi > 0$.

It is important to stress that the increase of temperature for an adiabatic process for homogeneous deformation (i.e. before the localization phenomenon occurs) is not large, but it suffices to superpose some small plastic softening effect of the material. This thermal plastic softening effect is crucial for a proper explanation of the mechanism of localization.

Of course, we can also introduce additional micro-damage mechanisms to describe realistic behaviour of metals (cf. Duszek and Perzyna, 1988a,b; Duszek–Perzyna *et al.*, 1990), but it seems that thermomechanical couplings and thermal softening effects have a dominating influence on the phenomenon of localization.

The results obtained are in agreement with experimental observations of the initiation of localization.

Anand and Spitzig (1980) performed experimental investigations of the initiation of localized shear bands in a maraging steel under the condition of quasi-static plane strain. They observed a small but essential difference between the values of the hardening modulus at localization for tension and compression.

From eqn (123) it follows that there is no influence of thermal plastic softening (represented by Π) on the direction of localization; however, the influence of thermal expansion (represented by Ξ) is rather distinct. For example, in the case of a pure shear stress state, we have $\tau_{II} = 0$, $\tau_I = -\tau_{III}$, then eqn (123) becomes

$$\tan^2\beta = \frac{\tau_I' + (1-2\nu)\Xi\sqrt{J_2}}{\tau_I' - (1-2\nu)\Xi\sqrt{J_2}} = \frac{T_{max} + (1-2\nu)\Xi}{T_{max} - (1-2\nu)\Xi}. \tag{146}$$

When material is elastically incompressible ($\nu = \frac{1}{2}$) or thermal expansion is neglected ($\Xi = 0$), eqn (146) yields $\beta = 45^\circ$, whereas taking into account the thermal expansion effect gives $\beta > 45^\circ$, which is in accordance with experimental observations. For states of stress other than pure shear the influence of the thermal expansion effect is such that deviation at the plane of localization from 45° is even more prominent.

6.2. *Influence of stress flux*

The matrix $\mathscr{H}$ arises in (134) due to the difference between L,τ and $\dot{\tau}$ and describes the influence of the spatial covariance terms on the condition for localization. This influence is effectively decribed in eqn (136) by H_1 or in eqn (139) by $\{\ \}$.

The matrix $\mathscr{H}$ consists of different terms which have the magnitude of a stress component divided by an elastic modulus G. These terms are generally small compared to unity. This justifies the approximation procedure assumed in the determination of H_{cr}/G [cf. eqn (139)].

However, it is reasonable to point out that the rate of hardening modulus H decreases in value with ongoing plastic deformation and all terms in H_0 become small and can be comparable to terms in H_1. This supports the conjecture that the influence of terms which arise from the difference between the Lie derivative and the material rate of the Kirchhoff stress tensor are important and may play a dominant role when the inception of localization phenomenon is expected to take place for small values of H, which is near to the maximum stress attained during the process.

In fact, the experimental results obtained for AISI 1018 cold rolled steel by Marchand *et al.* (1988), and performed by using a torsional Kolsky bar (split-Hopkinson bar) to impose a rapid deformation rate in a short thin-walled tubular specimen, showed that the inception of localization along the shear band takes place in the stage of the plastic deformation process when the range of nominal strains is about 15–45% and corresponds approximately to the maximum stress attained during the test. These results confirmed that with continued deformation the strain distribution is no longer homogeneous. During this stage of the process a continuous increase in the magnitude of localized strain along a narrow shear band is observed. As the nominal strain within this stage increases, the localized strain increases over 150%. In this stage of deformation the values of the hardening modulus rate and flow stress level do not vary greatly.

7. CONCLUSIONS

The analysis of the influence of thermomechanical couplings on adiabatic shear band localization criteria for finite elastic–plastic deformations based on the standard bifurcation theory has been presented.

Particular attention is focused on the objectivity of the rates and to the form of the constitutive and evolution equations proposed. A method has been developed which allows application of the standard bifurcation procedure in examination of the shear band localization criteria when plastic flow phenomena and thermomechanical coupling effects are taken into consideration.

The investigation of two important thermal effects, namely thermal expansion and thermal plastic softening, is described. It has been proved that thermal expansion implies that the inclination to instability for the axially symmetric compression is different from that for the axially symmetric tension. For tension the material is more sensitive to localization than for compression. This result is in agreement with experimental observations of the initiation of localized shear bands in maraging steel under the condition of quasi-static plane strain of Anand and Spitzig (1980). Thermal plastic softening makes the material more inclined to instability, independently of the state of stress. It has been shown that even a small increase of temperature for an adiabatic process implies the plastic softening effect which becomes crucial for the initiation of the mechanism of localization. This result coincides with the conclusion drawn from the experimental observations performed under dynamic loading conditions for steels reported by Hartley *et al.* (1987), Marchand and Duffy (1988) and Marchand *et al.* (1988).

Discussion of the influence of the covariance terms which arise from the difference between the Lee derivative and the material rate of the Kirchhoff stress tensor on the localization criteria is also presented. It has been shown that the influence of the covariance terms is important and may play a dominant role when the inception of localization phenomenon is expected to take place for small values of the rate of hardening modulus H, that is near to the maximum stress attained during the process. This result supports physically and experimentally justified conjecture.

REFERENCES

Abraham, R. and Mardsen, J. E. (1978). *Foundations of Mechanics*, 2nd Edn. Addison-Wesley, Reading, MA.

Abraham, R., Marsden, J. E. and Ratiu, T. (1988). *Manifolds, Tensor Analysis and Applications*. Springer, Berlin.

Anand, L. and Spitzig, W. A. (1980). Initiation of localized shear band in plane strain. *J. Mech. Phys. Solids* **28**, 113–128.

Duszek, M. K. and Perzyna, P. (1988a). Plasticity of damage solids and shear band localization. *Ing. Arch.* **58**, 380–392.

Duszek, M. K. and Perzyna, P. (1988). Influence of the kinematic hardening on the plastic flow localization in damaged solids. *Arch. Mech.* **40**, 595–609.

Duszek-Perzyna, M. K., Perzyna, P. and Stein, E. (1990). Adiabatic shear band localization in elastic–plastic damaged solids. The Second Int. Symposium on Plasticity and its Current Applications, 31 July–4 August 1989, Mie University, Tsu, Japan. *Int. J. Plasticity* (in press).

Hadamard, J. (1903). *Leçons sur la Propagation des Pndes et les Équations de l'Hydrodynamique*, Chap. 6, Paris.

Hartley, K. A., Duffy, J. and Havley, R. H. (1987). Measurement of the temperature profile during shear band formation in steels deforming at high strain rates. *J. Mech. Phys. Solids* **35**, 283–301.

Hill, R. (1962). Acceleration waves in solids. *J. Mech. Phys. Solids* **10**, 1–16.

Hill, R. and Rice, J. R. (1973). Elastic potentials and the structure of inelastic constitutive laws. *SIAM J. Appl. Math.* **25**, 448–461.

Lippmann, H. (1986). Velocity field equations and strain localization. *Int. J. Solids Structures* **22**, 1399–1409.

Mandel, J. (1966). Conditions de stabilité et postulat de Drucker. In *Rheology and Soil Mechanics* (Edited by J. Kravtchenko and P. M. Sirieys), pp. 58–68. Springer, Berlin.

Marchand, A. and Duffy, J. (1988). An experimental study of the formation process of adiabatic shear bands in a structural steel. *J. Mech. Phys. Solids* **36**, 251–283.

Marchand, A., Cho, K. and Duffy, J. (Sept. 1988). The formation of adiabatic shear bands in an AISI 1018 cold-rolled steel. Brown University Report.

Mardsen, J. E. and Hughes, T. J. R. (1983). *Mathematical Foundations of Elasticity*. Prentice-Hall, Englewood Cliffs, NJ.

Mear, M. E. and Hutchinson, J. E. (1985). Influence of yield surface curvature on flow localization in dilatant plasticity. *Mech. Mater.* **4**, 395–407.

Nemes, J. A., Eftis, J. and Randles, P. W. (1990). Viscoplastic constitutive modeling of high strain-rate deformation, material damage and spall fracture. *ASME J. Appl. Mech.* **57**, 282–291.

Needleman, A. and Rice, J. R. (1978). Limits to ductility set by plastic flow localization. In *Mechanics of Sheet Metal Forming* (Edited by D. P. Koistinen and N.-M. Wang), pp. 237–267. Plenum, New York.

Oldroyd, J. (1950). On the formulation of rheological equations of state. *Proc. R. Soc. Lond., Ser. A* **200**, 523–541.

Perzyna, P. (1988). Temperature and rate dependent theory of plasticity of crystalline solids. *Rev. Phys. Appl.* **23**, 445–459.

Perzyna, P. (1989). Constitutive equations of dynamic plasticity. *Post Symposium Short Course*, August 4–5, 1989. Nagoya, Japan.

Perzyna, P. (1990). Temperature and rate dependent theory of plasticity of polycrystalline solids. The Second Int. Symposium on Plasticity and its Current Applications, 31 July–4 August 1989, Mie University, Tsu, Japan. *Ing. Arch.* (in press).

Rice, J. R. (1971). Inelastic constitutive relations for solids: an internal variable theory and its application to metal plasticity. *J. Mech. Phy. Solids* **19**, 433–455.

Rice, J. R. (1973). Plasticity of soil mechanics. In *Proc. Symp. of the Role of Plasticity in Soil Mechanics* (Edited by A. C. Palmer), pp. 263–275. Cambridge, U.K.

Rice, J. R. (1976). The localization of plastic deformation. In *Theoretical and Applied Mechanics* (Edited by W. T. Koiter), pp. 207–220. North Holland, Amsterdam.

Rice, J. R. and Rudnicki, J. W. (1980). A note on some features of the theory of localization of deformation. *Int. J. Solids Structures* **16**, 597–605.

Rudnicki, J. W. and Rice, J. R. (1975). Conditions for the localization of deformation in pressure-sensitive dilatant materials. *J. Mech. Phys. Solids* **23**, 371–394.

Simo, J. C. (1988). A framework for finite strain elastoplasticity based on maximum plastic dissipation and the multiplicative decomposition, Part I. Continuum formulation. *Comput. Math. Appl. Mech. Engng* **66**, 199–219.

Thomas, T. Y. (1961). *Plastic Flow and Fracture in Solids*. Academic Press, New York.

Truesdell, C. and Noll, W. (1965). The non-linear field theories of mechanics. In *Handbuch der Physik* (Edited by S. Flügge), Vol. III/3, pp. 1–579. Springer, Berlin.

Tvergaard, V. (1987). Effect of yield surface curvature and void nucleation on plastic flow localization. *J. Mech. Phys. Solids* **35**, 43–60.